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Ray Analysis of Acoustic Field in a Fluid Layer Embedded in an Infinite Fluid

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***Abstract*:** An analytical method is developed,using ray theory, to determine the interior acoustic field in an embedded fluid layer excited by an obliquely incident plane acoustic wave. The magnitude and phase of the field are shown to be correct by comparison with wave theory results. The method can be extended to analyze the pressure field inside sonar domes.

Index Terms: *acoustic field; ray theory; wave theory; sonar domes;*

I. Introduction

The acoustic pressure field inside a sonar dome ensonified by a plane wave is of interest. The dome is comprised of a thin shell filled with water. In ships, the dome is filled with fresh-water. The dome is used to protect the sonar array inside the dome from physical damage and noise due to flow over the dome. The thickness of the dome is much less than the wavelength of the acoustic wave and the transmission coefficient is high. Therefore, it is neglected in this study. In this paper, ray theory is used to determine the acoustic field inside a thin fluid layer embedded in an infinite fluid. It is excited by a plane acoustic wave that is obliquely incident on it. The effect of multiple reflections and transmissions at the interfaces is included. Therefore, good agreement is obtained with results obtained using wave theory.

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Dynamic analysis of sonar domes can be done using FEM or BEM but the number of degrees of freedom is very large because the dome size is very much more than the wavelength in water. Therefore it is proposed to use ray theory to analyze the dome. Analysis of the thin fluid layer is a first step in this direction.

II. Thin Fluid Layer

A. Wave approach

Consider a thin fluid layer with boundaries at $x = a$ and$x = b$ as shown in Fig.1. A wave from an object that is very far away and is nearly plane when it reaches the layer (dome) is incident on the boundary at $x=a$. The incident wave is partly reflected and partly transmitted into the layer. The transmitted
waveis partly reflected and partly transmitted whenever it meets an interface. The pressure inside the layer is of interest.

The densities of the three fluidsin Fig. 1 are ρ_1,ρ_2 and ρ_3, respectively and the speeds of sound in them are c_1,c_2, and c_3, respectively.

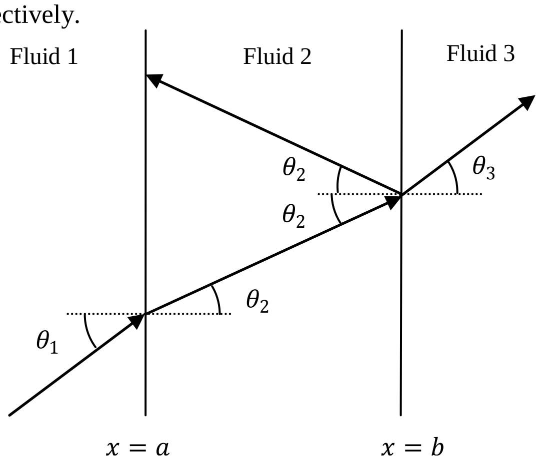

Fig. 1. Thin fluid layer at oblique incidence.

1) Governing Equations

Consider an acoustic wave incident obliquely on the fluid layer. The angular frequency of the wave is ω and it makes an angle θ_1 with respect to the x axis. Part of the wave is reflected at the interface $x = a$. Part of the wave gets transmitted through the layer and exits into fluid 3 at an angle θ_3. Within the layer, standing waves are generated by two waves: one travelling from left to right along the +x axis and another travelling from right to left along the –x axis.

The acoustic pressure in the incident wave and the reflected wave are expressed as (Kinsler *et al.*)

$$p_i = P_i e^{j(\omega t - \gamma_1 x - \zeta_1 y)} \qquad (1)$$

$$p_r = P_r e^{j(\omega t + \gamma_1 x - \zeta_1 y)} \qquad (2)$$

where t denotes time, and $k_\mathrm{i} = \omega/\mathrm{c_i}$, $k_\mathrm{i} \cos\theta_\mathrm{i} = \gamma_\mathrm{i}$, and $k_\mathrm{i} \sin\theta_\mathrm{i} = \zeta_\mathrm{i}$ for i = 1, 2, 3.

The pressures in the waves travelling to the right and to the left within the fluid layer are expressed as

$$p_a = A e^{j(\omega t - \gamma_2 x - \zeta_2 y)} \qquad (3)$$

$$p_b = B e^{j(\omega t + \gamma_2 x - \zeta_2 y)} \qquad (4)$$

respectively. The pressure in fluid 3 is expressed as:

$$p_t = \mathrm{P_t} e^{j(\omega t - \gamma_3 x - \zeta_3 y)} \qquad (5)$$

In the above, P_i is the amplitude of the incident wave, P_r is the amplitude of the reflected wave at the first interface, A and B are the amplitudes of the waves within the layer and P_t is the amplitude of the transmitted wave at the second interface

2) Continuity Conditions

When a wave passes through the interface between different media, pressure and velocity are continuous at the interface. These two continuity conditions, at the interfaces at $x = a$ and $x = b$, are used to determine the coefficients P_r, A, B, and P_t as described below.

Acoustic pressures on both sides of the boundary must be equal. At first interface i.e.at x=a,

$$p_i + p_r = p_a + p_b \qquad (6)$$

Substituting Eqs. (1) to (4) in (6) yields

$$-P_r e^{j\gamma_1 \mathrm{a}} + A e^{-j\gamma_2 \mathrm{a}} + B e^{j\gamma_2 \mathrm{a}} = P_i e^{-j\gamma_1 \mathrm{a}} \qquad (7)$$

At the second interface i.e. at x=b,

$$p_a + p_b = p_t \qquad (8)$$

Substituting Eqs. (3) to (5) in (8) yields

$$A e^{-j\gamma_2 \mathrm{b}} + B e^{j\gamma_2 \mathrm{b}} - P_t e^{j\gamma_3 \mathrm{b}} = 0 \qquad (9)$$

The normal components of the particle velocities on both sides of the boundary are equal. At the first interface i.e. x=a,

$$u_i + u_r = u_a + u_b \qquad (10)$$

The normal component of the particle velocity is given by,

$$u = \frac{p}{\rho c} \cos\theta \qquad (11)$$

On substituting for each of these the resulting equation will be,

$$P_r e^{j\gamma_1 a} + A\Re e^{-j\gamma_2 a} - B\Re e^{j\gamma_2 a} = P_i e^{-j\gamma_1 a} \qquad (12)$$

where $\Re = \frac{r_1}{r_2} \frac{\cos\theta_2}{\cos\theta_1}$.

At the second interface, i.e. at x=b,

$$u_a + u_b = u_t \quad (13)$$

The resulting equation is,

$$-Ae^{-j\gamma_2 b} + Be^{j\gamma_2 b} + P_t\mathcal{R}e^{j\gamma_3 b} = 0 \quad (14)$$

where $\mathcal{R} = \frac{r_2 \cos\theta_3}{r_3 \cos\theta_2}$.

Combining all the continuity conditions and expressing them in matrix form yields

$$\begin{bmatrix} -e^{j\gamma_1 a} & e^{-j\gamma_2 a} & e^{j\gamma_2 a} & 0 \\ 0 & e^{-j\gamma_2 b} & e^{j\gamma_2 b} & -e^{-j\gamma_3 b} \\ e^{j\gamma_1 a} & \Re e^{-j\gamma_2 a} & -\Re e^{j\gamma_2 a} & 0 \\ 0 & -e^{-j\gamma_2 b} & e^{j\gamma_2 b} & \mathcal{R}e^{-j\gamma_3 b} \end{bmatrix} \begin{Bmatrix} P_r \\ A \\ B \\ P_t \end{Bmatrix} = \begin{Bmatrix} e^{-j\gamma_1 a} \\ 0 \\ e^{-j\gamma_1 a} \\ 0 \end{Bmatrix} \quad (15)$$

This equation is easily solved analytically or by using MATLAB.

3) Special Cases

The equations obtained using the above general approach are solved for a few special cases. The answers that are obtained are expected and give confidence that the equations are correct.

1. Homogenous medium. For $r_1 = r_2 = r_3$ where $r_i = \rho_i c_i$, *i* = 1, 2, 3, the solution to Eq. (15) is P_r = 0, B = 0, A = 1,P_t= 1 for any angle of incidence. This is expected as there is no reflection when a wave propagates through a homogenous medium.
2. Normal incidence. For θ_1= 0, Kinsler et al (1982)[4] present the solution to Eq. (15). The same result is obtained by solving Eq. (15) for arbitrary values of r_i , *i* = 1, 2, 3.
3. Power. There is no power dissipation in the fluid. Therefore, the sum of the reflected and transmitted powers should be equal to the incident power. This is expressed as

$$R\pi + T\pi = 1 \quad (16)$$

where R_π is the power reflection coefficient and T_π is the power transmission coefficient. These coefficients are expressed as

$$R_\pi = |R|^2$$

$$T_\pi = \frac{A_t}{A_i}\frac{r_1}{r_2}|T|^2$$

where $R = P_r/P_i$ is the pressure reflection coefficient and $T = P_t/P_i$ is the pressure transmission coefficient. The power transmission coefficients are computed by using the solutions to Eq. (15) and it is found that Eq. (16) is valid in all cases.
4. Comparison with reflection and transmission coefficients explained by Brekhovskikh(1980)[1] for waves in layered media and found out that the results are similar

B. Ray approach

Consider a thin fluid layer with boundaries at x=a and x=b. The density of fluid above the interface, inside the fluid layer and below the interface is ρ_1,ρ_2 and $\rho_3$respectively and speed of sound in the fluid leftto the interface 1, inside the fluid layer and right to the interface 2 is c_1,$c_2$and $c_3$respectively.Here the fluid left and right to the layer is taken as having similar properties. So ρ_1=ρ_3 and c_1= c_3.

A ray is incident on the first interface i.e. at x=a of the fluid layer will get partially reflected and partially get transmitted. The transmitted ray again get reflected and transmitted whenever it meets the interface, and this process continues whenever the ray meets the interface. The total up-travelling signal is the sum of infinite number of partial reflections and transmissions. The pressure reflection and transmission coefficients are given by Clay and Medwin (1977)[2].

$$R_{12} = \frac{\rho_2 c_2 \cos\theta_1 - \rho_1 c_1 \cos\theta_2}{\rho_2 c_2 \cos\theta_1 + \rho_1 c_1 \cos\theta_2} \quad (17)$$

$$T_{12} = \frac{2\rho_2 c_2 \cos\theta_1}{\rho_2 c_2 \cos\theta_1 + \rho_1 c_1 \cos\theta_2} \quad (18)$$

$$R_{23} = \frac{\rho_3 c_3 \cos\theta_2 - \rho_2 c_2 \cos\theta_3}{\rho_3 c_3 \cos\theta_2 + \rho_2 c_2 \cos\theta_3} \quad (19)$$

$$T_{23} = \frac{2\rho_3 c_3 \cos\theta_2}{\rho_3 c_3 \cos\theta_2 + \rho_2 c_2 \cos\theta_3} \quad (20)$$

$$R_{12} = -R_{21} \quad (21)$$

$$T_{12}T_{21} = 1 - {R_{12}}^2 \quad (22)$$

Since the first and last medium are the same, $R_{12} = -R_{23}$

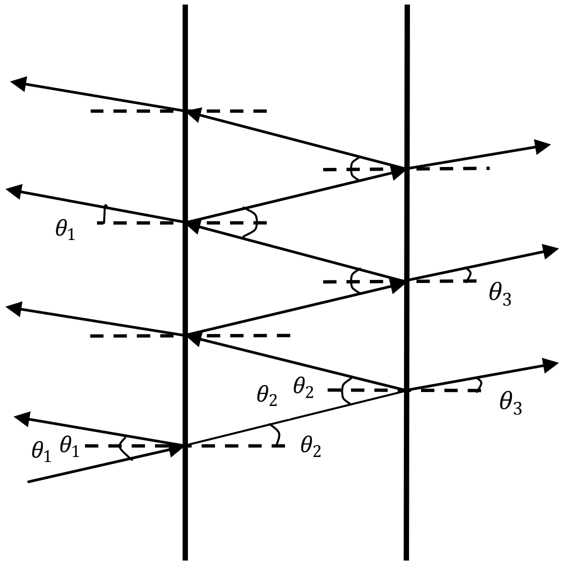

Fig.2. Traced rays inside the fluid layer with oblique incident ray

Considering the case with origin on the point of incidence at the first interface i.e. $x = a$ and second interface is at $x = b$, here taken $a=0$, $b=h$ where $h = b - a$.

Considering a wave front passing through the origin under consideration i.e. at the point of incidence, wave front is defined as the surface on which phase is constant. It therefore follows that the wave front, in 2D space, at any instant of time, is a straight line. Wave front is drawn perpendicular to the direction of travel of the rays. In general, the potential of nth ray is given by,

$$\emptyset_n = A_n e^{j(\omega t + k_1 \xi_n - \eta_n)} \quad (23)$$

where A_n represents the amplitude of the ray and $(\omega t + k_1 \xi_n - \eta_n)$ represents the phase of the ray. The time-dependence, $e^{j(\omega t)}$, is present in all the following equations but is suppressed for convenience, ξ_n is the path not travelled by the nth ray from the origin and η_n is the phase term indicating extra path travelled by the nth ray from the origin.Phase of the ray is expressed in a local coordinate system and the x axis of the local coordinate system is along the ray direction.

1) Reflection coefficient at the first interface

For calculating the reflection coefficient at the first interface, consider all the rays that are travelling back to fluid 1 from the first interface. i.e. R_1, R_2, R_3, R_4 from the Fig.3. R_1 is the ray which is reflected back to fluid 1 when incident ray strikes on the interface 1. R_2ray undergoes one transmission at interface 1 (to fluid 2),one reflection at interface 2 (in fluid 2),one transmission at interface 1 (to fluid 1).R_3 ray undergoes one transmission at interface 1 (to fluid 2),two reflections at interface 2 (in fluid 2), one reflection at interface 1 (in fluid 2),one transmission at interface 1 (to fluid 1).

Consider the wave front perpendicular to these rays which pass through the origin under consideration. In general the potential of the rays travelling back from the first interface can be written as, for nth ray,

$$\emptyset_{nar} = A_{nr} e^{j(\omega t + k_1 \xi_n - \eta_n)} \quad (24)$$

Total reflection is given by the sum of potentials of all the rays travelling back to fluid 1 from the first interface. i.e. taken as R_{13}

$$R_{13} = \emptyset_{R1} + \emptyset_{R2} + \emptyset_{R3} + \cdots \ldots \ldots \ldots \ldots \ldots \infty \quad (25)$$

where,

$\emptyset_{R1} = R_{12} e^{j(\omega t + k_1 \xi_0 - \eta_0)}$, potential corresponding to $R_{1,}$

$\emptyset_{R2} = T_{12} R_{23} T_{21}\, e^{j(\omega t + k_1 \xi_1 - \eta_1)}$,potential corresponding to $R_{2,}$

$\emptyset_{R3} = T_{12} {R_{23}}^2 T_{21}\, R_{21} e^{j(\omega t + k_1 \xi_2 - \eta_2)}$, potential corresponding to $R_{3,}$

The path not travelled by the ray R_1 from origin (shown as thicker lines) is given by, $\xi_0 = 0$, since the ray R_1 passes through origin. Path not travelled by R_2 from origin is obtained as,

$\xi_1 = 2h \tan\theta_2 \sin\theta_1$, by performing angle calculations.

Similarly for $R_{3,}$

$\xi_2 = 4h \tan\theta_2 \sin\theta_1$.

In general for an n^th ray,

$$\xi_n = 2h \tan\theta_2 \sin\theta_1\, n \quad (26)$$

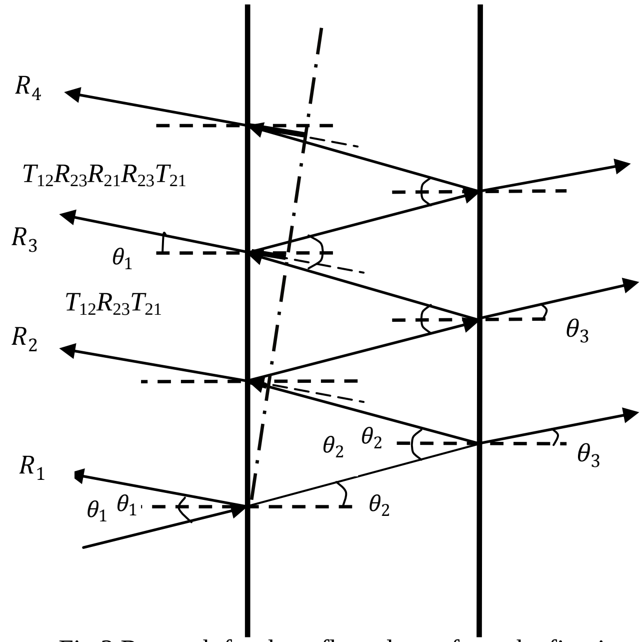

Fig.3 Ray path for the reflected rays from the first interface

The phase term indicating extra path travelled by the ray R_1 from origin is given by, $\eta_0 = 0$, since the ray R_1 passes through origin. Thephase term indicating extra distance travelled by R_2 from origin is obtained as, $\eta_1 = k_2 \frac{2h}{\cos\theta_2}$.Similarly for $R_{3,}$ $\eta_2 = k_2 \frac{4h}{\cos\theta_2}$.

In general for an n^th ray,

$$\eta_n = k_2 \frac{2h}{cos\,\theta_2} n \quad (27)$$

Combining these values, we can formulate a general expression,

$$k_1 \xi_{\boldsymbol{n}} - \eta_n = -2k_2 h \, cos\,\theta_2 n \quad (28)$$

The summation series above, will formulate a geometric series from the second term. The common ratio of the series is $R_{23} R_{21} e^{-j2k_2 h \cos\theta_2}$. Using the geometric series summation formulae we can derive a single expression which will give the value of R_{13}.

$$R_{13} = \frac{R_{12}+R_{23}e^{-j2k_2h\cos\theta_2}}{1+R_{12}R_{23}e^{-j2k_2h\cos\theta_2}} \qquad (29)$$

2) Transmission coefficient at the first interface

For calculating the transmission coefficient at the first interface, consider all the rays that are travelling to fluid 2 from the first interface, i.e. A_1, A_2, A_3, A_4 from the Fig 4. A_1 is the ray which is transmitted to fluid 2 when incident ray strikes on the interface 1. A_2 ray undergoes one transmission at interface 1 (to fluid 2),one reflection at interface 2 (in fluid 2),and one reflection at interface 1 (in fluid 2). A_3 ray undergoes one transmission at interface 1 (to fluid 2),two reflections at interface 2 (in fluid 2), two reflections at interface 1 (in fluid 2).

Consider the wave front perpendicular to these rays which pass through the origin under consideration. In general the potential of the rays travelling from the first interface to fluid 2 can be written as, for n^{th} ray,

$$\emptyset_{nat} = A_{nt}\,e^{j(\omega t+k_1\xi_n-\eta_n)} \qquad (30)$$

Total transmission is given by the sum of all potentials of all the transmitted rays from first interface to fluid 2 at the wave front, i.e. A_{12}.

$$A_{12} = \emptyset_{A1} + \emptyset_{A2} + \emptyset_{A3} + \cdots\ldots\ldots\ldots\ldots\infty \qquad (31)$$

where,

$\emptyset_{A1} = T_{12}e^{j(\omega t+k_1\xi_0-\eta_0)}$, potential corresponding to A_1,
$\emptyset_{A2} = T_{12}R_{23}R_{21}\,e^{j(\omega t+k_1\xi_1-\eta_1)}$, potential corresponding to A_2,
$\emptyset_{A3} = T_{12}R_{23}{}^2R_{21}{}^2e^{j(\omega t+k_1\xi_2-\eta_2)}$, potential corresponding to A_3,

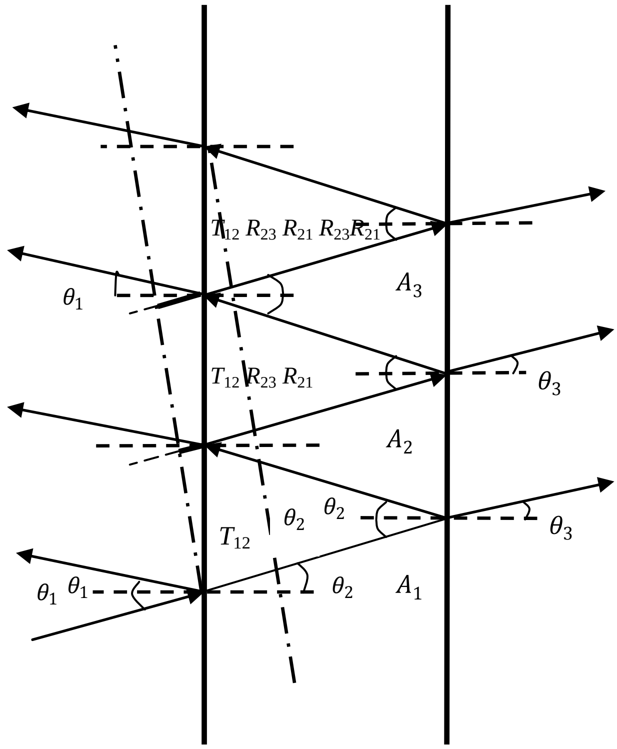

Fig.4 Ray path for the transmitted rays from the first interface

The path not travelled by the ray A_1 from origin (shown in dark lines) is given by, $\xi_0 = 0$, since the ray A_1 passes through origin. Path not travelled by A_2 from origin is obtained as,
$\xi_1 = 2h\tan\theta_2\sin\theta_1$, by performing some angle calculations.
Similarly for A_3,
$\xi_2 = 4h\tan\theta_2\sin\theta_1$.
In general for an n^{th} ray,

$$\xi_n = 2h\tan\theta_2\sin\theta_1\,n \qquad (32)$$

The phase term indicating extra path travelled by the ray A_1 from origin is given by, η_0 = 0, since the ray A_1 passes through origin. The phase term indicating extra distance travelled by A_2 from origin is obtained as, $\eta_1 = k_2 \frac{2h}{\cos\theta_2}$.Similarly for A_3, $\eta_2 = k_2 \frac{4h}{\cos\theta_2}$.
In general for an n^th ray,

$$\eta_n = k_2 \frac{2h}{\cos\theta_2} \text{n} \qquad (33)$$

Combining these values, we can formulate a general expression,

$$k_1\xi_n - \eta_n = -2k_2 \text{h} \cos\theta_2 \text{n} \qquad (34)$$

The summation series above, will formulate a geometric series from the first term. The common ratio of the series is $R_{23}R_{21}e^{-j2k_2h\cos\theta_2}$. Using the geometric series summation formulae we can derive a single expression which will give the value of A_{12}.

$$A_{12} = \frac{T_{12}}{1+R_{23}R_{12}e^{-j2k_2h\cos\theta_2}} \qquad (35)$$

3) Reflection coefficient at the second interface

For calculating the transmission coefficient at the second interface, consider all the rays that are travelling to fluid 2 from the second interface, i.e. B_1, B_2, B_3, B_4 from the Fig 5.B_1 is the ray which is reflected to fluid 2 after undergoing a transmission from interface1 when incident ray strikes on the interface 1. B_2 ray undergoes one transmission at interface 1 (to fluid 2), two reflections at interface 2 (in fluid 2),one reflection at interface 1 (in fluid 2). B_3 ray undergoes one transmission at interface 1 (to fluid 2),three reflections at interface 2 (in fluid 2), and two reflections at interface 1 (in fluid 2).

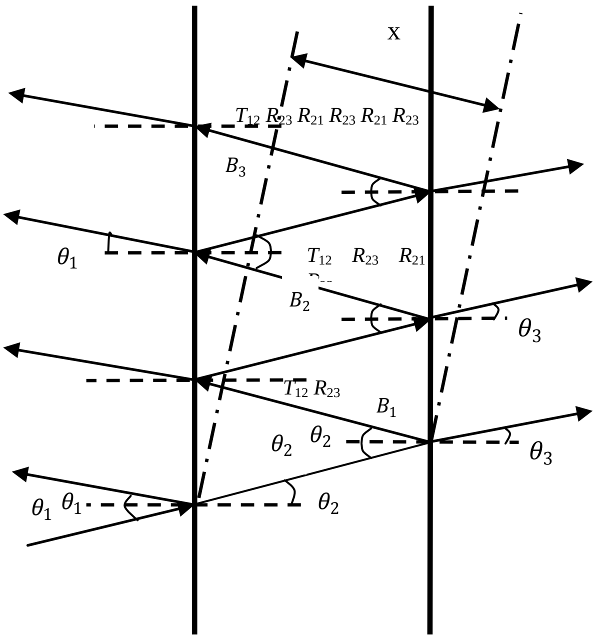

Fig.5 Ray path for the reflected rays from the second interface

Consider the wave front perpendicular to these rays passing through the first point of reflection in interface2. In general the potential of the rays travelling from the second interface to fluid 2 can be written as, for n^th ray,

$$\emptyset_{nbr} = B_{nr} e^{j(\omega t + k_2\xi_n + k_2 x - \eta_n)} \qquad (36)$$

Total reflection is given by the sum of all potentials of all the reflected rays from second interface to fluid 2 at the wave front, i.e. B_{23}.

$$B_{23} = \emptyset_{B1} + \emptyset_{B2} + \emptyset_{B3} + \cdots \ldots\ldots\ldots\ldots\ldots\infty \qquad (37)$$

where,
$\emptyset_{B1} = T_{12}R_{23}e^{j(\omega t + k_2\xi_0 + k_2 x - \eta_0)}$, potential corresponding to B_1
$\emptyset_{B2} = T_{12}R_{23}{}^2R_{21}\, e^{j(\omega t + k_2\xi_1 + k_2 x - \eta_1)}$,potential corresponding to B_2
$\emptyset_{B3} = T_{12}R_{23}{}^3R_{21}{}^2e^{j(\omega t + k_2\xi_2 + k_2 x - \eta_2)}$potential corresponding to B_3

Since in this case we considered the wave front at a local point i.e. at the first point of reflection, we need to shift it to the point of incidence of the ray at interface 1.For this we need to move the wave front parallel so that it should pass through the origin we considered in the above cases. On doing this there comes a constant shift of all points at which the rays intersects the wave front and that is denoted as x.The path not travelled by the ray B_1 from origin is given by,$\xi_0 = 0$, since the ray B_1 passes through origin. Path not travelled by B_2 from origin is obtained as,

$\xi_1 = 2h\tan\theta_2 \sin\theta_3 n$, by performing angle calculations.

Similarly for A_3,

$\xi_2 = 4h\tan\theta_2 \sin\theta_3 n$.

In general for an n^th ray,

$$\xi_n = 2h\tan\theta_2 \sin\theta_3 n \qquad (38)$$

The phase term indicating extra path travelled by the ray B_1 from origin is given by, $\eta_0 = \frac{k_2 h}{\cos\theta_2}$. The phase term indicating extra distance travelled by B_2 from origin is obtained as,

$\eta_1 = k_2 \frac{3h}{\cos\theta_2}$.Similarly for A_3, $\eta_2 = k_2 \frac{5h}{\cos\theta_2}$.

In general for an n^th ray,

$$\eta_n = \frac{(2n+1)k_2 h}{\cos\theta_2} \qquad (39)$$

x_0 can be computed by performing angle calculations and it is given by,

$$\mathrm{x} = \frac{h}{\cos\theta_2}\cos 2\theta_2 \qquad (40)$$

Combining these values, we can formulate a general expression,

$$k_2\xi_n + k_2\mathrm{x} - \eta_n = -2k_2 h\cos\theta_2\, n - \frac{k_2 h}{\cos\theta_2}(1+\cos 2\theta_2) \qquad (41)$$

The summation series above, will formulate a geometric series from the first term. The common ratio of the series is $R_{23}R_{21}e^{-j2k_2 h\cos\theta_2}$. Using the geometric series summation formulae we can derive a single expression which will give the value of B_{23}.

$$B_{23} = \frac{T_{12}R_{23}e^{-j\frac{k_2 h}{\cos\theta_2}(1+\cos 2\theta_2)}}{1+R_{23}R_{12}e^{-j2k_2 h\cos\theta_2}} \qquad (42)$$

4) Transmission coefficient at the second interface

For calculating the transmission coefficient at the second interface, consider all the rays that are travelling to fluid 3 from the second interface, i.e. T_1, T_2, T_3, T_4 from the Fig.6.

T_1 is the ray generated when incident ray strikes on the interface 1,getting transmitted to fluid 2, which is again transmitted to fluid 3. T_2 ray undergoes one transmission at interface 1 (to fluid 2),one reflection at interface 2 (in fluid 2),one reflection at interface 1 (in fluid 2),one transmission at interface 2 (to fluid 3). T_3 ray undergoes one transmission at interface 1 (to fluid 2),two reflections at interface 2 (in fluid 2), two reflections at interface 1 (in fluid 2), and one transmission at interface 2 (to fluid 3).

Consider the wave front perpendicular to these rays passing through the first point of transmission at interface2. In general the potential of the rays travelling from the second interface to fluid 3 can be written as, for n^th ray,

$$\emptyset_{nbt} = B_{nt}\, e^{j(\omega t + k_2\xi_n - k_3 x_0 - \eta_n)} \qquad (43)$$

Total transmission is given by the sum of all potentials of all the transmitted rays from second interface to fluid 3 at the wave front, i.e. T_{12}.

$$T_{12} = \emptyset_{T1} + \emptyset_{T2} + \emptyset_{T3} + \cdots \ldots\ldots\ldots\ldots\ldots \infty \qquad (44)$$

where,

$\emptyset_{B1} = T_{12}T_{23}e^{j(\omega t + k_2\xi_0 - k_3 x_0 - \eta_0)}$, potential corresponding to T_1

$\emptyset_{B2} = T_{12}R_{23}R_{21}T_{23}e^{j(\omega t + k_2\xi_1 - k_3 x_0 - \eta_1)}$,potential corresponding to T_2

$\emptyset_{B3} = T_{12}{R_{23}}^2{R_{21}}^2T_{23}e^{j(\omega t + k_2\xi_2 - k_3 x_0 - \eta_2)}$,potential corresponding to T_3

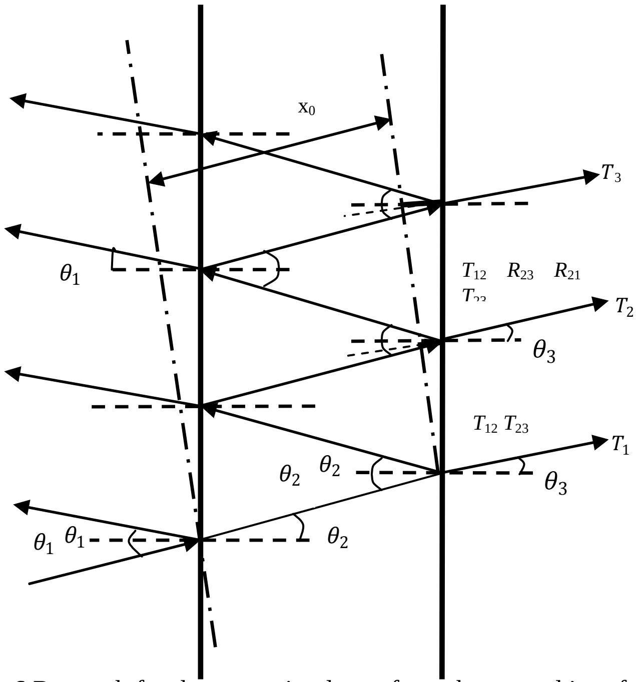

Fig.6 Ray path for the transmitted rays from the second interface

Since in this case we considered the wave front at a local point i.e. at the first point of transmission, we need to shift it to the point of incidence of the ray at interface 1.For this we need to move the wave front parallel so that it should pass through the origin we considered in the first two cases. On doing this there comes a constant shifts of all points at which the rays intersects the wave front and that is denoted as x_0.

The path not travelled by the ray T_1 from origin is given by, $\xi_0 = 0$, since the ray T_1 passes through origin. Path not travelled by T_2 from origin is obtained as,
$\xi_1 = 2h \tan\theta_2 \sin\theta_3 n$, by performing angle calculations.
Similarly for $T_{3,}$
$\xi_2 = 4h \tan\theta_2 \sin\theta_3 n$.
In general for an n[th] ray,

$$\xi_n = 2h \tan\theta_2 \sin\theta_3 n \qquad (45)$$

The phase term indicating extra path travelled by the ray T_1 from origin is given by, $\eta_0 = \frac{k_2 h}{\cos\theta_2}$. The phase term indicating extra distance travelled by T_2 from origin is obtained as,
$\eta_1 = k_2 \frac{3h}{\cos\theta_2}$.Similarly for $T_{3,}$ $\eta_2 = k_2 \frac{5h}{\cos\theta_2}$.
In general for an n[th] ray,

$$\eta_n = \frac{(2n+1)k_2 h}{\cos\theta_2} \qquad (46)$$

x_1 can be computed by performing angle calculations and it is given by,

$$x_0 = \frac{h}{\cos\theta_2}\cos(\theta_3 - \theta_2) \qquad (47)$$

Combining these values, we can formulate a general expression,

$$k_2\xi_n - k_3 x_0 - \eta_n = -2k_2 h \cos\theta_2\, n - k_2 \frac{h}{\cos\theta_2} + k_3 \frac{h}{\cos\theta_2}\cos(\theta_3 - \theta_2) \qquad (48)$$

The summation series above, will formulate a geometric series from the first term. The common ratio of the series is $R_{23}R_{21}e^{-j2k_2 h\cos\theta_2}$. Using the geometric series summation formulae we can derive a single expression which will give the value of T_{23}.

$$T_{13} = \frac{T_{12}T_{23}e^{-j\frac{h}{\cos\theta_2}(k_2 - k_3\cos(\theta_3-\theta_2))}}{1+R_{23}R_{21}e^{-j2k_2 h\cos\theta_2}} \qquad (49)$$

*C. **Numerical results and discussions***

Numerical results are presented to illustrate the pressure field for case of f = 10 kHz, P_i = 1 Pa, ρ_1 = 1000 kg/m3, c_1= 1500 m/s, ρ_2 = 1100 kg/m3, and c_2= 1650 m/s, $\rho_3 = \rho_1$, c_3=c_1 .The angle of incidence is 30 degrees. The interface is at x = 0 and x=1. Dots and lines are used to show wave theory and ray theory results. The continuous solid lines indicate the wave theory results and circled lines indicate ray theory results.

The values obtained from the wave approach P_r, A, B and P_t are compared with R_{12},A_{12},B_{23}, and T_{23} respectively and it is found that both the values are equal in phase and magnitude. This shows that the ray approach is accurate in this case.

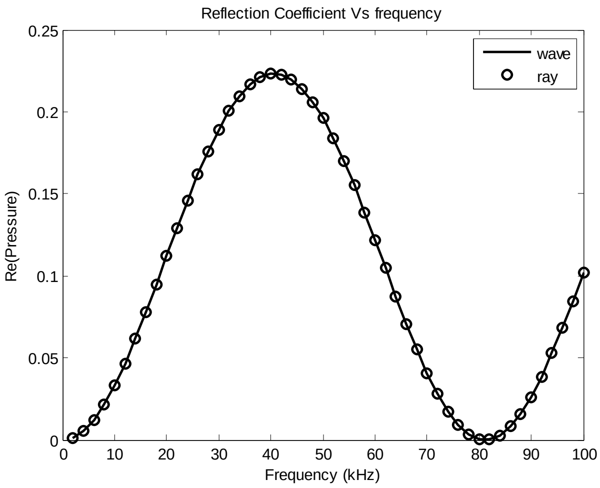

Fig 7 Variation of reflection coefficient(real part) at the first interface with the frequency

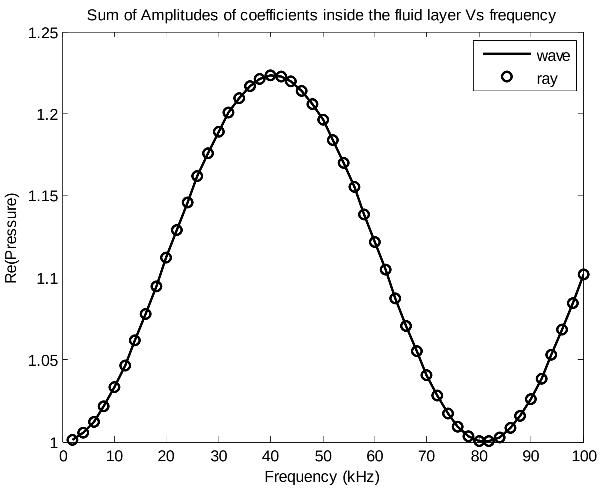

Fig 8 Variation of sum of the reflection and transmission coefficients (real part) inside the fluid layer with frequency

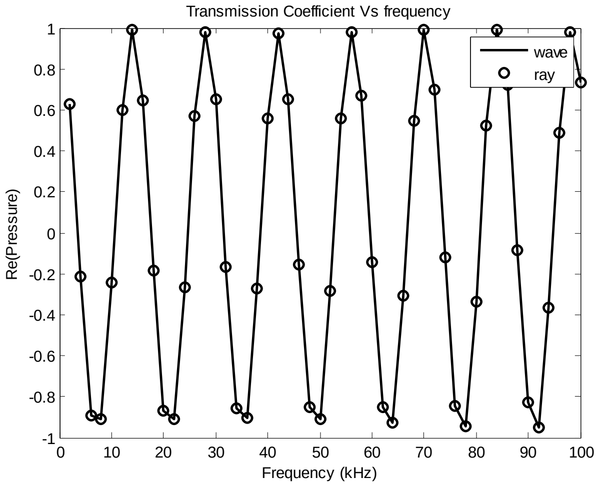

Fig 9 Variation of transmission coefficient (real part) at the second interface with the frequency

III. Conclusion

Our problem of interest is a Sonar dome which is comprised of a thin 3D shell filled with fresh water and operates at sea.As an initial problem, a simple case of 2D fluid layer is considered with interfaces at x=a and x=b and the reflection and transmission through the layer is found out using ray analysis. The phase term of the ray was not explained by Clay and Medwin[2] and his calculations were based on a local coordinate system. The results based on his approach didn't give good agreement with the wave analysis results. The phase of each ray is then computed in a global coordinate system which is then verified with wave approach.

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